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Adaptive fuzzy decentralized control for a class of interconnected nonlinear system with unmodeled dynamics and dead zones^{*}

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Abstract: In this paper, an adaptive decentralized fuzzy control scheme is presented for a class of multiple-input and multiple-output (MIMO) nonlinear interconnected systems with unmodeled dynamics and dead zones. In the controller design procedure, fuzzy logic systems are used to approximate the unknown nonlinear functions and then an adaptive fuzzy controller is developed by combining adaptive technique with backstepping. The proposed control scheme can guarantee that all signals in the closed-loop system are semi-globally uniformly ultimately bounded in mean square. Simulation results are provided to illustrate the effectiveness of the proposed scheme.

Key words: adaptive fuzzy control, nonlinear systems, unmodeled dynamics, backstepping.

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1 Introduction

In the past decades, the modeling and control of nonlinear systems have attracted considerable attention because of the ubiquity of nonlinearity. So far, some interesting control algorithms such as adaptive backstepping control [10], intelligent control [2–9] have been developed to deal with the control problems of nonlinear systems. Large-scale nonlinear systems, viewed as a set of interconnected subsystems, are existed widely in practical applications, such as robot control, servo applications, electric power systems [1]. Because of the physical configuration and high dimensionality of large-scale nonlinear interconnected systems, finding a centralized control scheme is a difficult task, neither economically feasible nor even necessary. Under these circumstances, decentralized control strategy only depending on local subsystem's state information is an efficient and effective way to achieve an objective for the whole large scale systems. Compared with centralized control scheme, the main advantages of decentralized control lie in that it can alleviate computational burden and enhance robustness and reliability against interacting operation failures. In [10], an adaptive backstepping decentralized control scheme is developed for large-scale interconnected nonlinear systems without satisfying the matching condition. Recently, there has been rapidly growing interest in fuzzy control of nonlinear systems, and there have been many meritorious results within the fuzzy control field [12–23]. Then, many interesting results on adaptive decentralized control are reported in [24–32] for interconnected systems with highly uncertain nonlinearities.

Dead-zone phenomenon, an important non-smooth nonlinearity in the actuator, exists in a wide range industrial processes, such as valves, DC servo motors, and other devices. The existence of dead zone input nonlinearity severely degrades system performance. So, the effect of dead zone nonlinearity cannot be ignored when controller is designed. Up to now, the control problem for nonlinear systems preceded by dead-zone nonlinearities has been an active topic, and there exist many remarkable results which have been reported in [33–37] and the references therein. An immediate approach for control of dead-zone is to design an adaptive dead-zone inverse, which was presented by Tao and Kokotovic in [33]. Further, in [34], Wang et al. proposed a robust adaptive control scheme without using the dead-zone inverse

technique. Then, by combining adaptive backstepping technique with fuzzy logic systems or neural networks, some approximation-based adaptive control algorithms have been reported in [35,37,38] for nonlinear systems with unknown dead-zone input. The aforementioned control schemes, however, only consider the nonlinear systems with the nonlinear uncertainties without considering the unmodeled dynamics and dynamic disturbances [39–41]. As stated in [43], unmodeled dynamics and dynamic disturbances, which frequently include in the practical systems because of the modeling errors, external disturbances, modeling simplifications or changes with time variations, make the control synthesis more difficult. Therefore, the robust adaptive control for nonlinear systems with unmodeled dynamics is very important both in the control theory and practical applications. In [39], a robust adaptive controller is proposed for a class of strict-feedback nonlinear systems with dynamic uncertainties. Liu and Li [40] developed an adaptive decentralized control scheme for nonlinear interconnected systems with unmodeled dynamics. Further, in [41], the problem of robust adaptive output-feedback control is considered for nonlinear systems with unmodeled dynamics. Recently, several interesting adaptive fuzzy (or neural networks) control of strict-feedback nonlinear systems with unmodeled dynamics and uncertain nonlinear functions are presented in [19,42] for state-feedback control and in [43] for output-feedback control, respectively. Although the control design of nonlinear systems with dynamic uncertainties has achieved a great progress, to the author’s best knowledge, few results are available for large-scale nonlinear interconnected systems with unmodeled dynamics and unknown dead-zone inputs which are common in the practical applications.

With the aforementioned observations, in this paper, we consider the problem of fuzzy adaptive decentralized control for a class of large-scale nonlinear interconnected systems with unmodeled dynamics, dynamics disturbances, nonlinear uncertainties and dead-zone input nonlinearities. By combining fuzzy systems’ universal approximation capability with adaptive backstepping technique, an adaptive fuzzy control algorithm is developed. It is shown that the proposed controller guarantees that all signals in the closed-loop system are semi-globally uniformly ultimately bounded in mean square. The main advantage of this paper is that only one adaptive parameter is involved in the proposed adaptive controller

for each subsystems. In this way, the computational burden is significantly alleviated.

The rest of the paper is organized as follows. In Section 2, the problem formulation and preliminaries are given. A backstepping-based adaptive fuzzy control scheme is presented in Section 3. A simulation example is given in Section 4, and followed by Section 5 which concludes the work.

2 Problem Formulation And Preliminaries

In this paper, we consider a class of MIMO interconnected nonlinear systems with unmodeled dynamics and input dead-zone nonlinearities. Its i th ($i = 1, 2, \dots, N$) subsystem is described as follows:

$$\begin{cases} \dot{z}_i = q_i(z_i, x_i), \\ \dot{x}_{ij} = g_{ij}(\underline{x}_{ij})x_{ij+1} + f_{ij}(\underline{x}_{ij}) + h_{ij}(\bar{y}) + \Delta_{ij}(\underline{x}_{ij}, z_i, t), j = 1, 2, \dots, n_i - 1, \\ \dot{x}_{in_i} = g_{in_i}(\underline{x}_{in_i})u_i + f_{in_i}(\underline{x}_{ij}) + h_{in_i}(\bar{y}) + \Delta_{in_i}(\underline{x}_{in_i}, z_i, t), \\ y_i = x_{i1}, \end{cases} \quad (1)$$

where $x_i = [x_{i1}, x_{i2}, \dots, x_{in_i}]^T \in R^{n_i}$ and $y_i \in R$ are the state vector and the scalar output of the i th nonlinear subsystem, respectively. To simplify the denotations, define $\underline{x}_{ij} = [x_{i1}, x_{i2}, \dots, x_{ij}]^T \in R^j$. $z_i \in R^{n_{i0}}$ in (1) denotes the unmeasured portion of the state. The z_i -dynamics in (1) is referred to as the unmodeled dynamics, $\Delta_{ij}(\cdot)$ is an uncertain dynamic disturbance, $g_{ij}(\cdot)$ are unknown smooth nonlinear function. $f_{ij}(\cdot) : R^{j+1} \rightarrow R$, ($j = 1, 2, \dots, n_i$) are unknown smooth nonlinear functions, $h_{ij}(\cdot) : R^N \rightarrow R$ ($j = 1, 2, \dots, n_i$) are unknown smooth interconnections between the i th subsystem and other subsystems, with $f_{ij}(0) = h_{ij}(0) = 0$. It is supposed that $\Delta_{ij}(\cdot)$ and $q_i(\cdot)$ in (1) are uncertain Lipschitz continuous functions. $u_i \in R$ is the output of an unknown dead zone [43] and defined as

$$u_i = D_i(v_i) = \begin{cases} m_{ir}(v_i), & v_i \geq b_{ir} \\ 0, & b_{il} < v_i < b_{ir} \\ m_{il}(v_i), & v_i \leq b_{il} \end{cases} \quad (2)$$

where $v_i \in R$ is the input of the dead-zone, $b_{il} < 0$ and $b_{ir} > 0$ are the unknown parameters.

The control objective of this paper is to design an adaptive fuzzy control scheme such that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded.

To facilitate the controller design, the following assumptions are required.

Assumption 1. [43] For the dynamic disturbances $\Delta_{ij}(i = 1, 2, \dots, N; j = 1, 2, \dots, n_i)$ in (1), there exist unknown nonnegative smooth functions $\phi_{ij1}(\cdot)$ and $\phi_{ij2}(\cdot)$, such that

$$|\Delta_{ij}| \leq \phi_{ij1}(|\underline{x}_{ij}|) + \phi_{ij2}(|z_i|). \quad (3)$$

Remark 1. This assumption is similar to the one in [43] in which $\phi_{ij1}(\cdot)$ and $\phi_{ij2}(\cdot)$ are known. Assumption 1, however, does not require them to be known. Therefore, Assumption 1 relaxes the restriction in the existing results.

Assumption 2 [31]. For uncertain nonlinear functions $h_{ij}(\bar{y})$ in (1), there exist unknown smooth functions $h_{ijl}(y_l)$ such that for $1 \leq i \leq N, 1 \leq j \leq n_i$,

$$|h_{ij}(\bar{y})|^2 \leq \sum_{l=1}^N h_{ijl}^2(y_l), \quad (4)$$

where $h_{ijl}(0) = 0, l = 1, 2, \dots, N$.

Remark 2. Noting $h_{ijl}(y_l)$ in Assumption 2 are smooth functions with $h_{ijl}(0) = 0$, so there exist unknown smooth functions $\bar{h}_{ijl}(y_l)$ such that

$$|h_{ij}(\bar{y})|^2 \leq \sum_{l=1}^N y_l^2 \bar{h}_{ijl}^2(y_l), \quad (5)$$

Assumption 3. [43] The unmodeled dynamics in (1) is exponentially input-to-state practically stable (exp-ISpS); i.e, for the system $\dot{z}_i = q_i(z_i, x_i)$, there exists an exp-ISpS Lyapunov function $V_i(z_i)$ such that

$$\alpha_{i1}(|z_i|) \leq V_i(z_i) \leq \alpha_{i2}(|z_i|), \quad (6)$$

$$\frac{\partial V_i(z_i)}{\partial z_i} q_i(z_i, x_i) \leq -c_i V_i(z_i) + \mu_i(|x_{i1}|) + d_i, \quad (7)$$

where α_{i1} , α_{i2} and μ_i are of class K_∞ -functions, c_i and d_i are known positive constants.

Assumption 4. [44] For $1 \leq j \leq n_i$, the signs of $g_{ij}(\underline{x}_{ij})$ are known, and there exists unknown positive constant b_m such that

$$0 < b_m \leq |g_{ij}(\underline{x}_{ij})| < \infty, \forall \bar{x}_{ij} \in R^{n_i}. \quad (8)$$

Assumption 5. [43] For smooth functions $m_{il}(v_i)$ and $m_{ir}(v_i)$, there exist unknown positive constants n_{il0} n_{il1} n_{ir0} and n_{ir1} such that

$$0 < n_{il0} \leq m'_{il}(v_i) \leq n_{il1}, \forall v_i \in (-\infty, b_{il}], \quad (9)$$

$$0 < n_{ir0} \leq m'_{ir}(v_i) \leq n_{ir1}, \forall v_i \in [b_{ir}, +\infty), \quad (10)$$

and $\gamma_{i0} \leq \min \{n_{il0}, n_{ir0}\}$ is an unknown positive constant, where $m'_{il}(v_i) = \frac{dm_{il}(s)}{ds} \big|_{s=v_i}$ and $m'_{ir}(v_i) = \frac{dm_{ir}(s)}{ds} \big|_{s=v_i}$

Based on Assumption 5, the dead-zone (2) can be written in the following form:

$$u_i = D_i(v_i) = \Theta_i^T(t) \Omega_i(t) v_i + d_i(v_i) \quad (11)$$

where $\Omega_i(t) = [\bar{\varphi}_{ir}(t), \bar{\varphi}_{il}(t)]^T$, $\Theta_i(t) = [n_{ir}(v_i(t)), n_{il}(v_i(t))]^T$.

$$\bar{\varphi}_{ir}(t) = \begin{cases} 1, & v_i(t) > b_{il} \\ 0, & v_i(t) \leq b_{il} \end{cases} \quad (12)$$

$$\bar{\varphi}_{il}(t) = \begin{cases} 1, & v_i(t) < b_{ir} \\ 0, & v_i(t) \geq b_{ir} \end{cases} \quad (13)$$

$$n_{ir}(v_i) = \begin{cases} 0, & v_i \leq b_{il} \\ m'_{ir}(\xi_{ir}(v_i)), & b_{il} < v_i < +\infty \end{cases} \quad (14)$$

$$n_{il}(v_i) = \begin{cases} m'_{il}(\xi_{il}(v_i)), & -\infty < v_i < b_{ir} \\ 0, & v_i \geq b_{ir} \end{cases} \quad (15)$$

$$d_i(v_i) = \begin{cases} -m'_{ir}(\xi_{ir}(v_i))b_{ir}, & v_i \geq b_{ir} \\ -[m'_{il}(\xi_{il}(v_i)) + m'_{ir}(\xi_{ir}(v_i))]v_i, & b_{il} < v_i < b_{ir} \\ -m'_{il}(\xi_{il}(v_i))b_{il}, & v_i \leq b_{il} \end{cases} \quad (16)$$

where $\xi_{il}(v_i) \in (v_i, b_{il})$, if $v_i < b_{il}$; $\xi_{il}(v_i) \in (b_{il}, v_i)$, if $b_{il} \leq v_i < b_{ir}$; $\xi_{ir}(v_i) \in (b_{ir}, v_i)$, if $b_{ir} < v_i$; $\xi_{ir}(v_i) \in (v_i, b_{ir})$, if $b_{il} \leq v_i < b_{ir}$, and $|d_i(v_i)| \leq \delta_i^*$. Here δ_i^* is an unknown positive constant defined by $\delta_i^* = (n_{ir1} + n_{il1}) \max \{b_{ir}, -b_{il}\}$. In addition, $\theta_i^T(t) \Omega_i(t)$ satisfies $\theta_i^T(t) \Omega_i(t) \in [\gamma_{i0}, n_{il1} + n_{ir1}] \subset (0, +\infty)$.

Remark 3. Assumption 4 means that $g_{ij}(\cdot)$ are strictly either positive or negative. Without loss of generality, it is assumed that $0 < b_m \leq g_{ij}(\cdot)$. In addition, by means of Assump-

tions 4 and 5, it can be further supposed that

$$0 < b \leq g_{ij}(\cdot), 1 \leq i \leq N, 1 \leq j \leq n_i - 1, 0 < b \leq g_{in_i}(\cdot)\theta_i^T(t)\Omega_i(t), \quad (17)$$

where $b = \min \{b_m, b_m \gamma_{i0}\}$ is an unknown constant.

In what follows, fuzzy logic system will be used to approximate a continuous function $f(x)$ defined on some compact sets. Adopt the singleton fuzzifier, the product inference, and the center-average defuzzifier to deduce the following fuzzy rules:

R_i : IF x_1 is F_1^i and x_2 is F_2^i and \dots and x_n is F_n^i

Then y is B^i ($i=1, 2, \dots, N$),

where $x = [x_1, \dots, x_n]^T \in R^n$ and $y \in R$ are the input and output of the fuzzy system, respectively. F_i^j and B^i are fuzzy sets in R . Since the strategy of singleton fuzzification, center-average defuzzification and product inference are used, the output of the fuzzy system can be formulated as

$$y(x) = \frac{\sum_{j=1}^N \bar{W}_j \prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N [\prod_{i=1}^n \mu_{F_i^j}(x_i)]},$$

where \bar{W}_j is the point at which fuzzy membership function $\mu_{B^j}(\bar{W}_j)$ achieves its maximum value, which is assumed to be 1. Let

$$s_j(x) = \frac{\prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N [\prod_{i=1}^n \mu_{F_i^j}(x_i)]},$$

$S(x) = [s_1(x), \dots, s_N(x)]^T$ and $W = [\bar{W}_1, \dots, \bar{W}_N]^T$. Then the fuzzy logic system can be rewritten as

$$y(x) = W^T S(x). \quad (18)$$

If all memberships are chosen as Gaussian functions, the following lemma holds.

Lemma 1. [46] Let $f(X)$ be a continuous function defined on a compact set Ω . Then, for any given constant $\epsilon > 0$, there exists a fuzzy logic system (3) such that

$$\sup_{X \in \Omega} |f(X) - W^T S(X)| \leq \epsilon. \quad (19)$$

In the next section, a backstepping-based adaptive control procedure will be proposed. During the controller design, fuzzy logic system $W_{ij}^T S(X_{ij})$ will be used to model the packaged

unknown function $\bar{f}_{ij}(X_{ij})$ at Step j . Both the virtual control signals and adaption laws will be constructed in the following forms:

$$\alpha_{ij}(\bar{x}_{ij}) = -(\lambda_{ij} + 0.5)\bar{x}_{ij} - \frac{1}{2a_{ij}^2}\bar{x}_{ij}\hat{\theta}_i S_{ij}^T(X_{ij})S_{ij}(X_{ij}), \quad (20)$$

$$\dot{\hat{\theta}}_i = \sum_{j=1}^{n_i} \frac{\gamma_i}{2a_{ij}^2} \bar{x}_{ij}^2 S_{ij}^T(X_{ij})S_{ij}(X_{ij}) - \sigma_i \hat{\theta}_i, \quad (21)$$

where, for $1 \leq i \leq N$, $1 \leq j \leq n_i$, λ_{ij} , a_{ij} , γ_i and σ_i are positive design parameters, $X_{ij} = [\underline{x}_{ij}^T, \hat{\theta}_i, r_i]^T$ with $\underline{x}_{ij} = [x_{i1}, x_{i2}, \dots, x_{ij}]^T$, and \bar{x}_{ij} satisfies the following coordinate transformation:

$$\bar{x}_{ij} = x_{ij} - \alpha_{i(j-1)}, \quad (22)$$

where $\alpha_{i0} = 0$, $\hat{\theta}_i$ is the estimation of unknown constant θ_i which is defined as

$$\theta_i = \frac{1}{b} \|W_{ij}\|^2; 1 \leq i \leq N, 1 \leq j \leq n_i, \quad (23)$$

where $\|W_{ij}\|$ denotes the norm of the ideal weight vector of fuzzy logic systems, which will be specified at the j th design step. Specifically, α_{in_i} denotes the actual control input v_i .

Remark 4. Obviously, the inequality (21) implies that for any nonnegative initial condition $\hat{\theta}_i(t_0) \geq 0$, the solution $\hat{\theta}_i(t) \geq 0$ holds for $t \geq 0$. therefore, throughout this paper, it is assumed that $\hat{\theta}_i(t) \geq 0$.

Lemma 2. [45] For any variable $\eta \in R$ and constant $\epsilon > 0$, the following inequality holds:

$$0 \leq |\eta| - \eta \tanh\left(\frac{\eta}{\epsilon}\right) \leq \delta\epsilon, \delta = 0.2785. \quad (24)$$

Lemma 3. [43] If V_i is an exp-ISpS Lyapunov function for a control system, i.e, Eqs. (6) and (7) hold, then for any constants \bar{c}_i in $(0, c_{i0})$, any initial condition $x_{i0} = x_{i0}(0)$, and any function $\bar{\mu}(x_{i1}) \geq \mu(|x_{i1}|)$ there exists finite time $T_{i0} = T_{i0}(\bar{c}_i, r_{i0}, z_{i0})$, nonnegative function $D_i(t)$ defined for all $t \geq 0$ and a signal described by

$$\dot{r}_i = -\bar{c}_i r_i + \bar{\mu}_i(x_{i1}(t)) + d_i, r_i(0) = r_{i0}, \quad (25)$$

such that $D_i(t) = 0$ for all $t \geq T_{i0}$,

$$V_i(z_i(t)) \leq r_i(t) + D_i(t). \quad (26)$$

For all $t \geq 0$, where the solutions are defined. Without losing of generality, this paper takes $\bar{\mu}_i(\cdot)$ as $\bar{\mu}_i(s) = s^2 \mu_{i0}(s^2)$, where $\bar{\mu}_i(\cdot)$ is a nonnegative smooth function. Therefore, the dynamical r_i defined by (25) becomes

$$\dot{r}_i = -\bar{c}_i r_i + x_{i1}^2 \mu_{i0}(|x_{i1}^2|) + d_{i0}, r_i(0) = r_{i0}, \quad (27)$$

where μ_{i0} is a nonnegative smooth function.

3 Adaptive fuzzy control design

This section develops fuzzy backstepping for a class of MIMO nonlinear systems. For simplicity, the time variable t and the state vector \underline{x}_{ij} will be omitted from the corresponding functions and let $S_{ij}(X_{ij}) = S_{ij}$.

Step 1. Base on $\bar{x}_{i1} = x_{i1}$, let us first consider the subsystem

$$\begin{aligned} \dot{z}_i &= q_i(z_i, x_i), \\ \dot{x}_{i1} &= g_{i1}x_{i2} + f_{i1} + h_{i1}(\bar{y}) + \Delta_{i1}(x_{i1}, z_i, t). \end{aligned} \quad (28)$$

To stabilize the subsystem (28), we consider a Lyapunov function as

$$V_{i1} = \frac{1}{2} \bar{x}_{i1}^2 + \frac{1}{\lambda_{i0}} \dot{r}_i + \frac{b}{2\gamma_i} \tilde{\theta}_i^2. \quad (29)$$

Then, based on (25), the time derivative of V_{i1} along the solution of (28) is

$$\begin{aligned} \dot{V}_{i1} &= \bar{x}_{i1} \dot{\bar{x}}_{i1} + \frac{1}{\lambda_{i0}} \dot{r}_i - \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i \\ &= \bar{x}_{i1} (g_{i1}x_{i2} + f_{i1} + h_{i1}(\bar{y}) + \Delta_{i1}) + \frac{1}{\lambda_{i0}} \dot{r}_i - \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i \\ &\leq \bar{x}_{i1} (g_{i1}x_{i2} + f_{i1} + h_{i1}(\bar{y})) + |\bar{x}_{i1}| \phi_{i11}(|x_{i1}|) + |\bar{x}_{i1}| \phi_{i12}(|z_i|) - \frac{\bar{c}_i}{\lambda_{i0}} r_i \\ &\quad + \frac{1}{\lambda_{i0}} (x_{i1}^2 \mu_{i0}(|x_{i1}^2|) + d_{i0}) - \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i, \end{aligned} \quad (30)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

With the help of (5) and the completion of squares, one has

$$\begin{aligned} \bar{x}_{i1} h_{i1}(\bar{y}) &\leq \frac{1}{2} \bar{x}_{i1}^2 + \frac{1}{2} h_{i1}^2(\bar{y}) \\ &\leq \frac{1}{2} \bar{x}_{i1}^2 + \frac{1}{2} \sum_{l=1}^N y_l^2 \bar{h}_{il}^2(y_l). \end{aligned} \quad (31)$$

By using Assumption 1 and Lemma 2, we can obtain

$$\begin{aligned}
|\bar{x}_{i1}|\phi_{i11}(|x_{i1}|) &= |\bar{x}_{i1}|\phi_{i11}(|x_{i1}|) - \bar{x}_{i1}\phi_{i11}(|x_{i1}|) \tanh\left(\frac{\bar{x}_{i1}\phi_{i11}(|x_{i1}|)}{\epsilon_{i11}}\right) \\
&\quad + \bar{x}_{i1}\phi_{i11}(|x_{i1}|) \tanh\left(\frac{\bar{x}_{i1}\phi_{i11}(|x_{i1}|)}{\epsilon_{i11}}\right) \\
&\leq \epsilon'_{i11} + \bar{x}_{i1}\phi_{i11}(|x_{i1}|) \tanh\left(\frac{\bar{x}_{i1}\phi_{i11}(|x_{i1}|)}{\epsilon_{i11}}\right) \\
&\leq \bar{x}_{i1}\hat{\phi}_{i11}(x_{i1}) + \epsilon'_{i11},
\end{aligned} \tag{32}$$

where $\epsilon'_{i11} = 0.2785\epsilon_{i11}$ and $\hat{\phi}_{i11}(x_{i1}) = \phi_{i11}(|x_{i1}|) \tanh(\frac{\bar{x}_{i1}\phi_{i11}(|x_{i1}|)}{\epsilon_{i11}})$ is a smooth nonlinear function.

By using the same derivations as [?], the following result holds:

$$\begin{aligned}
|\bar{x}_{i1}|\phi_{i12}(|z_i|) &\leq |\bar{x}_{i1}|\bar{\phi}_{i12}(r_i) + \frac{1}{4}\bar{x}_{i1}^2 + d_{i1}(t) \\
&\leq \bar{x}_{i1}\hat{\phi}_{i12}(x_{i1}, r_i) + \epsilon'_{i12} + \frac{1}{4}\bar{x}_{i1}^2 + d_{i1}(t),
\end{aligned} \tag{33}$$

where $\epsilon'_{i12} = 0.2785\epsilon_{i12}$, $d_{i1}(t) = (\phi_{i12} \circ \alpha_{i1}^{-1}(2D_i(t)))^2$ and $\hat{\phi}_{i12}(x_{i1}, r_i) = \bar{\phi}_{i12}(r_i) \tanh(\frac{\bar{x}_{i1}\bar{\phi}_{i12}(r_i)}{\epsilon_{i12}})$ with $\bar{\phi}_{i12}(r_i) = \phi_{i12} \circ \alpha_{i1}^{-1}(2r_i)$.

Substituting (32) and (33) into (30) results in

$$\begin{aligned}
\dot{V}_{i1} &\leq \bar{x}_{i1} \left(g_{i1}x_{i2} + f_{i1} + \frac{3}{4}\bar{x}_{i1} + \hat{\phi}_{i11}(x_{i1}) + \hat{\phi}_{i12}(x_{i1}, r_i) + \frac{1}{\lambda_{i0}}\bar{x}_{i1}\mu_{i0}(|\bar{x}_{i1}^2|) \right) \\
&\quad + \frac{1}{2} \sum_{l=1}^N y_l^2 \bar{h}_{i1l}^2(y_l) - \frac{\bar{c}_i}{\lambda_{i0}}r_i + \frac{d_{i0}}{\lambda_{i0}} + \sum_{k=1}^2 \epsilon'_{i1k} + d_{i1}(t) - \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\hat{\theta}}_i
\end{aligned} \tag{34}$$

Step j. ($2 \leq j \leq n_i - 1$) Similar procedures are taken for $i = 2, \dots, n - 1$ as those outlined in Step 1. The dynamics of $\bar{x}_{ij} = x_{ij} - \alpha_{i(j-1)}$ is given by

$$\dot{\bar{x}}_{ij} = g_{ij}x_{i(j+1)} + f_{ij} + h_{ij}(\bar{y}) + \Delta_{ij} - \dot{\alpha}_{i(j-1)} \tag{35}$$

where

$$\dot{\alpha}_{i(j-1)} = \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} (g_{ik}x_{i(k+1)} + f_{ik} + h_{ik}(\bar{y}) + \Delta_{ik}) + \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i + \frac{\partial \alpha_{i(j-1)}}{\partial r_i} \dot{r}_i \tag{36}$$

Consider a Lyapunov function candidate as

$$V_{ij} = \frac{1}{2} \bar{x}_{ij}^2 \tag{37}$$

Then, differentiating V_{ij} gives

$$\begin{aligned} \dot{V}_{ij} \leq & \bar{x}_{ij} \left(g_{ij} x_{i(j+1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{ij} + h_{ij}(\bar{y}) - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} \right. \\ & \left. - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} h_{ik}(\bar{y}) - \frac{\partial \alpha_{i(j-1)}}{\partial r_i} \dot{r}_i \right) + |\bar{x}_{ij} \bar{\Delta}_{ij}| - \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \end{aligned} \quad (38)$$

where $\bar{\Delta}_{ij} = \Delta_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} \Delta_{ik}$.

Following the same line as the procedures used in (31), one has

$$-\bar{x}_{ij} \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} h_{ik}(\bar{y}) \leq \frac{1}{2} \bar{x}_{ij}^2 \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} \right)^2 + \frac{1}{2} \sum_{k=1}^{j-1} \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l), \quad (39)$$

$$\bar{x}_{ij} h_{ij}(\bar{y}) \leq \frac{1}{2} \bar{x}_{ij}^2 + \frac{1}{2} \sum_{l=1}^N y_l^2 \bar{h}_{ijl}^2(y_l). \quad (40)$$

By using triangular inequality and Assumption 1, one has

$$\begin{aligned} |\bar{x}_{ij} \bar{\Delta}_{ij}| & \leq |\bar{x}_{ij}| (|\Delta_{ij}| + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} \right| |\Delta_{ik}|) \\ & \leq |\bar{x}_{ij}| \left(\phi_{ij1}(|\underline{x}_{ij}|) + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik1}(|\underline{x}_{ik}|) \right) \\ & \quad + |\bar{x}_{ij}| \left(\phi_{ij2}(|z_i|) + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik2}(|z_i|) \right). \end{aligned} \quad (41)$$

Further, similar to (32) and (33), we obtain

$$|\bar{x}_{ij}| \left(\phi_{ij1}(|\underline{x}_{ij}|) + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik1}(|\underline{x}_{ik}|) \right) \leq \bar{x}_{ij} \hat{\phi}_{ij1}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \epsilon'_{ij1} \quad (42)$$

$$\begin{aligned} |\bar{x}_{ij}| \left(\phi_{ij2}(|z_i|) + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik2}(|z_i|) \right) & \leq \bar{x}_{ij} \hat{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \epsilon'_{ij2} + d_{ij}(t) \\ & \quad + \frac{\bar{x}_{ij}^2}{4} \left[1 + \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right)^2 \right] \end{aligned} \quad (43)$$

where $\hat{\phi}_{ij1}(\underline{x}_{ij}, \hat{\theta}_i, r_i) = (\phi_{ij1} + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik1}) \tanh\left(\frac{\bar{x}_{ij}(\phi_{ij1} + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik1})}{\epsilon_{ij1}}\right)$, $\epsilon'_{ij1} = 0.2785 \epsilon_{ij1}$, $\hat{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) = \bar{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) \tanh\left(\frac{\bar{x}_{ij} \bar{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i)}{\epsilon_{ij2}}\right)$, $\bar{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) = \phi_{ij2} \circ \alpha_{i1}^{-1}(2r_i) + \sum_{k=1}^{j-1} \left| \frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right| \phi_{ik2} \circ \alpha_{i1}^{-1}(2r_i)$, $\epsilon'_{ij2} = 0.2785 \epsilon_{ij2}$, $d_{ij}(t) = \sum_{k=1}^j (\phi_{ik2} \circ \alpha_{i1}^{-1}(2D_i(t)))^2$, noting that $d_{ij}(t) \geq 0$ for all $t \geq 0$.

Subsequently, with the help of (41)-(43), we rewrite (38) as

$$\begin{aligned}
\dot{V}_{ij} = & \bar{x}_{ij} \left(g_{ij} x_{i(j+1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} + \hat{\phi}_{ij1}(\underline{x}_{ij}, \hat{\theta}_i, r_i) \right. \\
& + \hat{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{ij}}{4} \left[1 + \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right)^2 \right] - \frac{\partial \alpha_{i(j-1)}}{\partial r_i} \dot{r}_i \Big) \\
& + \frac{1}{2} \sum_{k=1}^j \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l) + d_{ij}(t) + \sum_{k=1}^2 \epsilon'_{ijk} - \bar{x}_{ij} \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i.
\end{aligned} \tag{44}$$

Step n_i : This is the n_i step, and the actual control law v_i is designed. Based on $\bar{x}_{in_i} = x_{in_i} - \alpha_{i(n_i-1)}$, then the time derivative of \bar{x}_{in_i} is

$$\dot{\bar{x}}_{in_i} = g_{in_i} u_i + f_{in_i} + h_{in_i}(\bar{y}) + \Delta_{in_i} - \dot{\alpha}_{i(n_i-1)} \tag{45}$$

where $\dot{\alpha}_{i(n_i-1)}$ has been given in (36) with $j = n_i$.

Choose a Lyapunov function candidate as

$$V_{in_i} = \frac{1}{2} \bar{x}_{in_i}^2. \tag{46}$$

Further, the dynamics of V_{in_i} satisfies

$$\begin{aligned}
\dot{V}_{in_i} \leq & \bar{x}_{in_i} \left(g_{in_i} (\Theta_i^T \Omega_i v_i + d_i(v_i)) + f_{in_i} + h_{in_i}(\bar{y}) - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} \right. \\
& - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} f_{ik} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} h_{ik}(\bar{y}) - \frac{\partial \alpha_{i(n_i-1)}}{\partial r_i} \dot{r}_i \Big) \\
& + |\bar{x}_{in_i} \bar{\Delta}_{in_i}| - \bar{x}_{in_i} \frac{\partial \alpha_{i(n_i-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i,
\end{aligned} \tag{47}$$

where $\bar{\Delta}_{in_i} = \Delta_{in_i} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} \Delta_{ik}$.

Then, by repeating the the similar derivations outlined from (41) to (44), we have

$$\begin{aligned}
\dot{V}_{in_i} \leq & \left(g_{in_i} (\Theta_i^T \Omega_i v_i + d_i(v_i)) + f_{in_i} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} f_{ik} \right. \\
& + \hat{\phi}_{in_i1}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) + \hat{\phi}_{in_i2}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{in_i}}{4} \left[1 + \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i(n_i-1)}}{\partial x_k} \right)^2 \right] \\
& - \frac{\partial \alpha_{i(n_i-1)}}{\partial r_i} \dot{r}_i \Big) + \frac{1}{2} \sum_{k=1}^{n_i} \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l) + d_{in_i}(t) + \sum_{k=1}^2 \epsilon'_{in_ik} \\
& - \bar{x}_{in_i} \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i
\end{aligned} \tag{48}$$

Now, choose the following Lyapunov function for the whole systems:

$$V = \sum_{i=1}^N \sum_{j=1}^{n_i} V_{ij} = \sum_{i=1}^N \sum_{j=1}^{n_i} \frac{1}{2} \bar{x}_{ij}^2 + \frac{b}{2\gamma_i} \tilde{\theta}_i^2. \quad (49)$$

By considering the fact of (34), (44) and (48), we obtain

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \bar{x}_{i1} \left\{ g_{i1} x_{i2} + f_{i1} + \frac{3}{4} \bar{x}_{i1} + \hat{\phi}_{i11}(x_{i1}) + \hat{\phi}_{i12}(x_{i1}, r_i) + \frac{1}{\lambda_{i0}} \bar{x}_{i1} \mu_{i0}(|\bar{x}_{i1}^2|) \right\} \\ & + \sum_{i=1}^N \sum_{j=2}^{n_i-1} \bar{x}_{ij} \left\{ g_{ij} x_{i(j+1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} \right. \\ & \left. + \hat{\phi}_{ij1}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \hat{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{ij}}{4} \left[1 + \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right)^2 \right] - \frac{\partial \alpha_{i(j-1)}}{\partial r_i} \dot{r}_i \right\} \\ & + \sum_{i=1}^N \bar{x}_{in_i} \left\{ g_{in_i} (\Theta_i^T \Omega_i v_i + d_i(v_i)) - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{in_i} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} f_{ik} \right. \\ & \left. + \hat{\phi}_{in_i1}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) + \hat{\phi}_{in_i2}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{in_i}}{4} \left[1 + \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i(n_i-1)}}{\partial x_k} \right)^2 \right] - \frac{\partial \alpha_{i(n_i-1)}}{\partial r_i} \dot{r}_i \right\} \\ & + \frac{1}{2} \sum_{i=1}^N \sum_{s=1}^{n_i} \sum_{k=1}^s \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l) - \frac{1}{\lambda_{i0}} \bar{c}_i r_i + \frac{1}{\lambda_{i0}} d_{i0} + \sum_{k=1}^2 \sum_{i=1}^N \sum_{j=1}^{n_i} \epsilon'_{ijk} + \sum_{i=1}^N \sum_{j=1}^{n_i} d_{ij}(t) \\ & - \sum_{i=1}^N \sum_{j=2}^{n_i} \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i - \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\hat{\theta}}_i. \end{aligned} \quad (50)$$

For the last term $\frac{1}{2} \sum_{i=1}^N \sum_{s=1}^{n_i} \sum_{k=1}^s \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l)$ in (50), by rearranging the sequence, it follows that

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^N \sum_{s=1}^{n_i} \sum_{k=1}^s \sum_{l=1}^N y_l^2 \bar{h}_{ikl}^2(y_l) &= \frac{1}{2} \sum_{i=1}^N \sum_{l=1}^N \sum_{s=1}^{n_i} \sum_{k=1}^s y_i^2 \bar{h}_{lki}^2(y_i) \\ &= \frac{1}{2} \sum_{i=1}^N \sum_{l=1}^N \sum_{s=1}^{n_l} \sum_{k=1}^s \bar{x}_{i1}^2 \bar{h}_{lki}^2(\bar{x}_{i1}). \end{aligned} \quad (51)$$

Utilizing adaptive law in (21) and rearranging terms in the summation results in

$$\begin{aligned} - \sum_{i=1}^N \sum_{j=2}^{n_i} \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i &= - \sum_{i=1}^N \sum_{j=2}^{n_i} \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \left(\sum_{k=1}^{n_i} \frac{\gamma_i}{2a_{ik}^2} \bar{x}_{ik}^2 S_{ik}^T S_{ik} - \sigma_i \hat{\theta}_i \right) \\ &\leq \sum_{i=1}^N \sum_{j=2}^{n_i} \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \sigma_i \hat{\theta}_i - \sum_{i=1}^N \sum_{j=2}^{n_i} \bar{x}_{ij} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \sum_{k=1}^{j-1} \frac{\gamma_i}{2a_{ik}^2} \bar{x}_{ik}^2 S_{ik}^T S_{ik} \\ &\quad + \sum_{i=1}^N \sum_{j=2}^{n_i} \frac{\gamma_i}{2a_{ij}^2} \bar{x}_{ij}^2 S_{ij}^T S_{ij} \sum_{k=2}^j |\bar{x}_{ik} \frac{\partial \alpha_{i(k-1)}}{\partial \hat{\theta}_i}| \end{aligned} \quad (52)$$

By taking $|d_i(v_i)| \leq \delta_i^*$ into account, we have

$$\bar{x}_{in_i} g_{in_i} d_i(v_i) \leq \frac{1}{2} \bar{x}_{in_i}^2 g_{in_i}^2 + \frac{1}{2} \delta_i^{*2}. \quad (53)$$

Substituting (51) - (53) into (50) yields

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \bar{x}_{i1} \left\{ g_{i1} \alpha_{i1} + \hat{f}_{i1}(X_{i1}) \right\} + \sum_{i=1}^N \sum_{j=2}^{n_i-1} \bar{x}_{ij} \left\{ g_{ij} \alpha_{ij} + \hat{f}_{ij}(X_{ij}) \right\} \\ & + \sum_{i=1}^N \bar{x}_{in_i} \left\{ g_{in_i} \Theta_i^T \Omega_i v_i + \hat{f}_{in_i}(X_{in_i}) \right\} - \sum_{i=1}^N \frac{\bar{c}_i}{\lambda_{i0}} r_i + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} \\ & + \sum_{i=1}^N \frac{1}{2} \delta_i^{*2} + \sum_{k=1}^2 \sum_{i=1}^N \sum_{j=1}^{n_i} \epsilon'_{ijk} + \sum_{i=1}^N \sum_{j=1}^{n_i} d_{ij}(t) - \sum_{i=1}^N \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\theta}_i \end{aligned} \quad (54)$$

where the functions $\bar{f}_{ij}(X_{ij})$, $1 \leq i \leq N, 1 \leq j \leq n_i$ are defined as

$$\begin{aligned} \hat{f}_{i1}(X_{i1}) = & f_{i1} + \frac{3}{4} \bar{x}_{i1} + \hat{\phi}_{i11}(x_{i1}) + \hat{\phi}_{i12}(x_{i1}, r_i) + \frac{\bar{x}_{i1}}{2} \sum_{l=1}^N \sum_{s=1}^{n_l} \sum_{k=1}^s \bar{h}_{lki}^2(\bar{x}_{i1}) \\ & + \frac{1}{\lambda_{i0}} \bar{x}_{i1} \mu_{i0}(|\bar{x}_{i1}^2|), \end{aligned} \quad (55)$$

$$\begin{aligned} \hat{f}_{ij}(X_{ij}) = & g_{i(j-1)} \bar{x}_{i(j-1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} + \hat{\phi}_{ij1}(\underline{x}_{ij}, \hat{\theta}_i, r_i) \\ & + \hat{\phi}_{ij2}(\underline{x}_{ij}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{ij}}{4} \left[1 + \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i(j-1)}}{\partial x_k} \right)^2 \right] + \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \sigma_i \hat{\theta}_i - \frac{\partial \alpha_{i(j-1)}}{\partial r_i} \dot{r}_i \\ & - \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i} \sum_{k=1}^{j-1} \frac{\gamma_i}{2a_{ik}^2} \bar{x}_{ik}^2 S_{ik}^T S_{ik} + \frac{\gamma_i}{2a_{ij}^2} \bar{x}_{ij} S_{ij}^T S_{ij} \sum_{k=2}^j |\bar{x}_{ik} \frac{\partial \alpha_{i(k-1)}}{\partial \hat{\theta}_i}|, \end{aligned} \quad (56)$$

$$\begin{aligned} \hat{f}_{in_i}(X_{in_i}) = & \frac{1}{2} \bar{x}_{in_i} g_{in_i}^2 - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} g_{ik} x_{i(k+1)} + f_{in_i} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} f_{ik} + \hat{\phi}_{in_i1}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) \\ & + \hat{\phi}_{in_i2}(\underline{x}_{in_i}, \hat{\theta}_i, r_i) + \frac{3\bar{x}_{in_i}}{4} \left[1 + \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i(n_i-1)}}{\partial x_k} \right)^2 \right] + \frac{\partial \alpha_{i(n_i-1)}}{\partial \hat{\theta}_i} \sigma_i \hat{\theta}_i - \frac{\partial \alpha_{i(n_i-1)}}{\partial r_i} \dot{r}_i \\ & - \frac{\partial \alpha_{i(n_i-1)}}{\partial \hat{\theta}_i} \sum_{k=1}^{n_i-1} \frac{\gamma_i}{2a_{ik}^2} \bar{x}_{ik}^2 S_{ik}^T S_{ik} + \frac{\gamma_i}{2a_{in_i}^2} \bar{x}_{in_i} S_{in_i}^T S_{in_i} \sum_{k=2}^{n_i} |\bar{x}_{ik} \frac{\partial \alpha_{i(k-1)}}{\partial \hat{\theta}_i}|. \end{aligned} \quad (57)$$

Since $f_{ij}, g_{ij}, \hat{\phi}_{ij1}(\cdot)$ and $\hat{\phi}_{ij2}(\cdot)$ are unknown smooth nonlinear functions, $\hat{f}_{ij}(X_{ij})$ cannot be directly used to construct a virtual control signal α_{ij} or the actual control input v_i . Fortunately, Lemma 1 guarantees that for any given $\varepsilon_{ij} \geq 0$ there exists a fuzzy logic system $W_{ij}^T S_{ij}(X_{ij})$ such that

$$\hat{f}_{ij}(X_{ij}) = W_{ij}^T S_{ij}(X_{ij}) + \delta_{ij}(X_{ij}), \quad (58)$$

where δ_{ij} refers to the approximation error and satisfies $|\delta_{ij}| < \varepsilon_{ij}$. Furthermore, by Young's inequality, one has

$$\begin{aligned}\bar{x}_{ij}\hat{f}_{ij}(X_{ij}) &= \bar{x}_{ij}\frac{W_{ij}^T}{\|W_{ij}\|}S_{ij}\|W_{ij}\| + \bar{x}_{ij}\delta_{ij} \\ &\leq \frac{1}{2a_{ij}^2}\bar{x}_{ij}^2\|W_{ij}\|^2S_{ij}^TS_{ij} + \frac{1}{2}a_{ij}^2 + \frac{b}{2}\bar{x}_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 \\ &\leq \frac{b}{2a_{ij}^2}\bar{x}_{ij}^2\theta_iS_{ij}^TS_{ij} + \frac{1}{2}a_{ij}^2 + \frac{b}{2}\bar{x}_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2,\end{aligned}\tag{59}$$

where the unknown constant θ_i has been defined in (23).

Substituting (58) into (54) and using (59) produces

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^N \bar{x}_{i1} \left(g_{i1}\alpha_{i1} + \frac{b}{2a_{i1}^2}\bar{x}_{i1}\theta_iS_{i1}^TS_{i1} \right) + \sum_{i=1}^N \sum_{j=2}^{n_i-1} \bar{x}_{ij} \left(g_{ij}\alpha_{ij} + \frac{b}{2a_{ij}^2}\bar{x}_{ij}\theta_iS_{ij}^TS_{ij} \right) \\ &\quad + \sum_{i=1}^N \bar{x}_{in_i} \left(g_{in_i}\Theta_i^T\Omega_i v_i + \frac{b}{2a_{in_i}^2}\bar{x}_{in_i}\theta_iS_{in_i}^TS_{in_i} \right) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^{n_i} \frac{b}{2}\bar{x}_{ij}^2 + \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\sum_{k=1}^2 \epsilon'_{ijk} + \frac{1}{2}a_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 + d_{ij}(t) \right) \\ &\quad + \frac{1}{2} \sum_{i=1}^N \delta_i^{*2} + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} - \sum_{i=1}^N \frac{1}{\lambda_{i0}} \bar{c}_i r_i - \sum_{i=1}^N \frac{b}{\gamma_i} \tilde{\theta}_i \dot{\theta}_i,\end{aligned}\tag{60}$$

At the present stage, we construct the virtual control signal α_{ij} in (20) and real controller v_i in (20) with $j = n_i$. Then, by applying the fact of (17), the following results hold:

$$\bar{x}_{ij}g_{ij}\alpha_{ij} \leq -\lambda_{ij}b\bar{x}_{ij}^2 - \frac{b}{2}\bar{x}_{ij}^2 - \frac{b}{2a_{ij}^2}\bar{x}_{ij}^2\hat{\theta}_iS_{ij}^TS_{ij},\tag{61}$$

$$\bar{x}_{in_i}g_{in_i}\theta_i^T\Omega_i v_i \leq -\lambda_{in_i}b\bar{x}_{in_i}^2 - \frac{b}{2}\bar{x}_{in_i}^2 - \frac{b}{2a_{in_i}^2}\bar{x}_{in_i}^2\hat{\theta}_iS_{in_i}^TS_{in_i}.\tag{62}$$

Then, by combining (60) together with (61) and (62), and using the adaptive law in (23), we rewrite (60) as

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^N \bar{x}_{i1} \left(-\lambda_{i1}b\bar{x}_{i1} + \frac{b}{2a_{i1}^2}\bar{x}_{i1}\tilde{\theta}_iS_{i1}^TS_{i1} \right) + \sum_{i=1}^N \sum_{j=2}^{n_i-1} \bar{x}_{ij} \left(-\lambda_{ij}b\bar{x}_{ij} + \frac{b}{2a_{ij}^2}\bar{x}_{ij}\tilde{\theta}_iS_{ij}^TS_{ij} \right) \\ &\quad + \sum_{i=1}^N \bar{x}_{in_i} \left(-\lambda_{in_i}b\bar{x}_{in_i} + \frac{b}{2a_{in_i}^2}\bar{x}_{in_i}\tilde{\theta}_iS_{in_i}^TS_{in_i} \right) - \sum_{i=1}^N \sum_{j=1}^{n_i} \frac{b}{\gamma_i} \tilde{\theta}_i \left(\frac{\gamma_i}{2a_{ij}^2}\bar{x}_{ij}^2S_{ij}^TS_{ij} - \sigma_i\hat{\theta}_i \right) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\sum_{k=1}^2 \epsilon'_{ijk} + \frac{1}{2}a_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 + d_{ij}(t) \right) + \sum_{i=1}^N \frac{1}{2}\delta_i^{*2} + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} - \sum_{i=1}^N \frac{\bar{c}_i}{\lambda_{i0}} r_i\end{aligned}$$

$$\begin{aligned}
&\leq -\sum_{i=1}^N \sum_{j=1}^{n_i} \left(\lambda_{ij} b \bar{x}_{ij}^2 - \frac{\sigma_i b}{\gamma_i} \tilde{\theta}_i \hat{\theta}_i \right) + \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\sum_{k=1}^2 \epsilon'_{ijk} + \frac{1}{2} a_{ij}^2 + \frac{1}{2b} \varepsilon_{ij}^2 + d_{ij}(t) \right) \\
&\quad + \frac{1}{2} \sum_{i=1}^N \delta_i^{*2} + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} - \sum_{i=1}^N \frac{\bar{c}_i}{\lambda_{i0}} r_i \\
&\leq -\sum_{i=1}^N \left(\sum_{j=1}^{n_i} \lambda_{ij} b \bar{x}_{ij}^2 + \frac{\sigma_i b}{2\gamma_i} \tilde{\theta}_i^2 + \frac{\bar{c}_i}{\lambda_{i0}} r_i \right) + \frac{1}{2} \sum_{i=1}^N \delta_i^{*2} + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} \\
&\quad + \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\sum_{k=1}^2 \epsilon'_{ijk} + \frac{1}{2} a_{ij}^2 + \frac{1}{2b} \varepsilon_{ij}^2 + d_{ij}(t) + \frac{\sigma_i b}{2\gamma_i} \theta_i^2 \right), \tag{63}
\end{aligned}$$

where the result $\tilde{\theta}_i \hat{\theta}_i \leq -\frac{1}{2} \tilde{\theta}_i^2 + \frac{1}{2} \theta_i^2$ has been used in the above inequality.

Theorem 1. Under Assumptions 1-4, consider the closed-loop nonlinear system consisted of the system (1), unknown dead zone nonlinearities (2), controller (20), and adaptive law (21). Then, under the action of controller (20), for any initial conditions $[x_{ij}^T(0), \hat{\theta}_i(0)]^T \in \Omega_0$ (where Ω_0 is an appropriately chosen compact set), all the signals in the closed-loop system are semi-globally uniformly ultimately bounded in the sense of mean square.

Proof: Let $a_0 = \min\{2\lambda_{ij}b, \bar{c}_i, \sigma_i, 1 \leq i \leq N, 1 \leq j \leq n_i\}$ and $b_0 = \frac{1}{2} \sum_{i=1}^N \delta_i^{*2} + \sum_{i=1}^N \frac{d_{i0}}{\lambda_{i0}} + \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\sum_{k=1}^2 \epsilon'_{ijk} + \frac{1}{2} a_{ij}^2 + \frac{1}{2b} \varepsilon_{ij}^2 + d_{ij}(t) + \frac{\sigma_i b}{2\gamma_i} \theta_i^2 \right)$. Then, we rewrite (63) in the following form:

$$\dot{V} \leq -a_0 V + b_0, \forall t \geq 0. \tag{64}$$

Further, multiplying (64) by $e^{-a_0 t}$ and integrating it over $[0, t]$ gives

$$V \leq (V(0) - \frac{b_0}{a_0}) e^{-a_0 t} + \frac{b_0}{a_0}, \forall t \geq 0 \tag{65}$$

which means that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded in mean square.

4 Simulation example

In this section, one numerical example is used to illustrate the effectiveness of the proposed control scheme.

Example 1. Consider the interconnected nonlinear system with dead zones and unmod-

eled dynamics as

$$\left\{ \begin{array}{l} \dot{z}_1 = -z_1 + 0.5x_{11}^2 + 0.5, \\ \dot{x}_{11} = (1 + x_{11}^2)x_{12} + x_{11}^2 \sin(x_{11}) + y_1 \ln(1 + y_2^2) + z_1 x_{11} \sin(x_{11}), \\ \dot{x}_{12} = u_1 + x_{11}x_{12} + y_2 \sin(y_1) + z_1 x_{11}x_{12}, \\ y_1 = x_{11}, \\ \dot{z}_2 = -z_2 + 0.5x_{21}^2 + 0.5, \\ \dot{x}_{21} = (2 + \sin(x_{21}))x_{22} + x_{21}x_{22} + y_1^2 y_2 + z_2 x_{21} \sin(x_{21}), \\ \dot{x}_{22} = u_2 + x_{21} \sin(x_{22}) + y_1 \cos(y_2^2) + z_2 x_{21} \sin(x_{22}), \\ y_2 = x_{21}, \end{array} \right. \quad (66)$$

where $u_i = D(v_i)$ is defined as

$$u_1 = D(v_1) = \begin{cases} 1.5(v_1 - 1.5), & v_1 \geq 1.5, \\ 0, & -1 < v_1 < 1.5, \\ 0.5(v_1 + 1), & v_1 \leq -1, \end{cases} \quad u_2 = D(v_2) = \begin{cases} 0.8(v_2 - 1), & v_2 \geq 1, \\ 0, & -0.5 < v_2 < 1, \\ 1.2(v_2 + 0.5), & v_2 \leq -0.5, \end{cases}$$

where x_{11} , x_{12} , x_{21} and x_{22} are the state variables, and y_i is the system output, u_i and v_i are the output and input of the dead-zone nonlinearity, respectively. It is obvious that the system (66) satisfies Assumptions 1-2 and 4-5. The control objective is to design an adaptive fuzzy controller such that all signals in the closed-loop system are bounded. Then, we check Assumption 3 holds for z_i -subsystem in (66), consider $V_i(z_i) = z_i^2$, then

$$\begin{aligned} \dot{V}_i(z_i) &= 2z_i(-z_i + 0.5x_{i1}^2 + 0.5) \\ &\leq -2z_i^2 + \frac{1}{4\varepsilon_i}z_i^2 + \varepsilon_i x_{i1}^4 + \frac{\varepsilon_i}{4} + \frac{z_i^2}{\varepsilon_i}. \end{aligned}$$

By choosing $\varepsilon_i = 2.5$, we have

$$\dot{V}_i(z_i) \leq -1.5z_i^2 + 2.5x_{i1}^4 + 0.625.$$

By choosing $\alpha_{i1}(|z_i|) = 0.5z_i^2$, $\alpha_{i2}(|z_i|) = 2z_i^2$, $c_{i0} = 1.2$, $d_{i0} = 0.625$ and $\mu(|x_{i1}|) = 2.5x_{i1}^4$, Assumption 3 is satisfied. Taking $\bar{c}_i = 1 \in (0, c_{i0})$ and defining the dynamic signal as follows:

$$\dot{r}_i = -r_i + 2.5x_{i1}^4 + 0.625. \quad (67)$$

By using Theorem 1, the virtual control law, the actual controller, and the adaptive laws are constructed as

$$\alpha_{i1} = -(\lambda_{i1} + 0.5)\bar{x}_{i1} - \frac{1}{2a_{i1}^2}\bar{x}_{i1}\hat{\theta}_i S_{i1}^T S_{i1}, \quad (68)$$

$$u_i = -(\lambda_{i2} + 0.5)\bar{x}_{i2} - \frac{1}{2a_{i2}^2}\bar{x}_{i2}\hat{\theta}_i S_{i2}^T S_{i2}, \quad (69)$$

$$\dot{\hat{\theta}}_i = \sum_{j=1}^2 \frac{\gamma_i}{2a_{ij}^2} \bar{x}_{ij}^2 S_{ij}^T S_{ij} - \sigma_i \hat{\theta}_i. \quad (70)$$

In simulation, the initial values are set to be $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0), z_1(0), z_2(0)]^T = [0.5, -0.2, 0.3, -0.3, 0, 0]^T$, $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$. The simulation is run by choosing the design constants as $\lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = 9$, $a_{11} = a_{12} = a_{21} = a_{22} = 2$, $\gamma_1 = \gamma_2 = 2$ and $\sigma_1 = \sigma_2 = 1$.

Then, Figs. 1-5 show the simulation results. Figure 1 shows the trajectories of state x_{11} and x_{12} and the trajectories of state x_{21} and x_{22} are displayed in Figure 2. Figures 3 and 4 depict the input signals u_1, v_1, u_2 and v_2 . The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are shown in Figure 5. Apparently, we can see that the proposed control approach can guarantee the boundedness of all signals in the closed-loop system.

5 Conclusion

This paper has proposed a robust adaptive fuzzy backstepping control approach for a class of large-scale systems with unmodeled dynamics, dynamics disturbances and input dead-zone nonlinearities. The proposed controller guarantees that all the signals in the closed-loop system remain semi-globally uniformly ultimately bounded in the sense of mean square. Simulation results have been provided to illustrate the effectiveness of the proposed control scheme. However, some further problems such as how to deal with tracking control problem of the original systems (1) with time delay by using the proposed method remain not to be considered. Therefore, the future research direction turns to tracking control problem.

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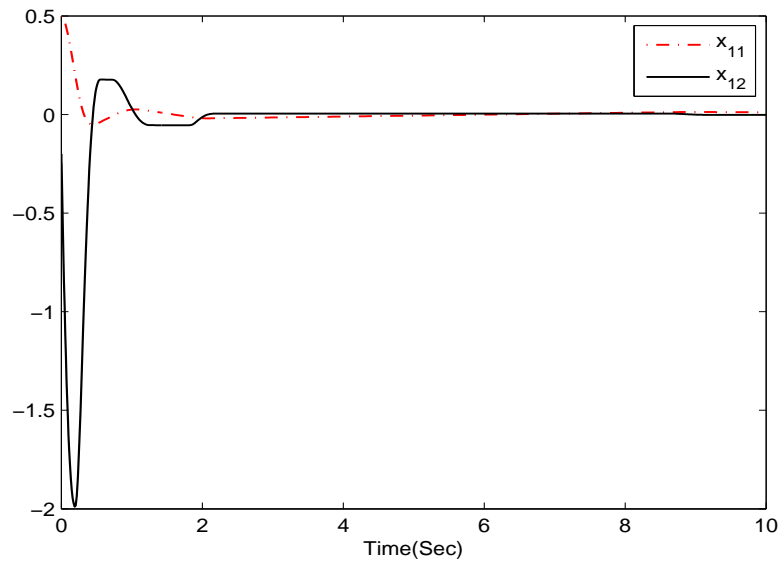


Fig. 1. State variables x_{11} and x_{12} .

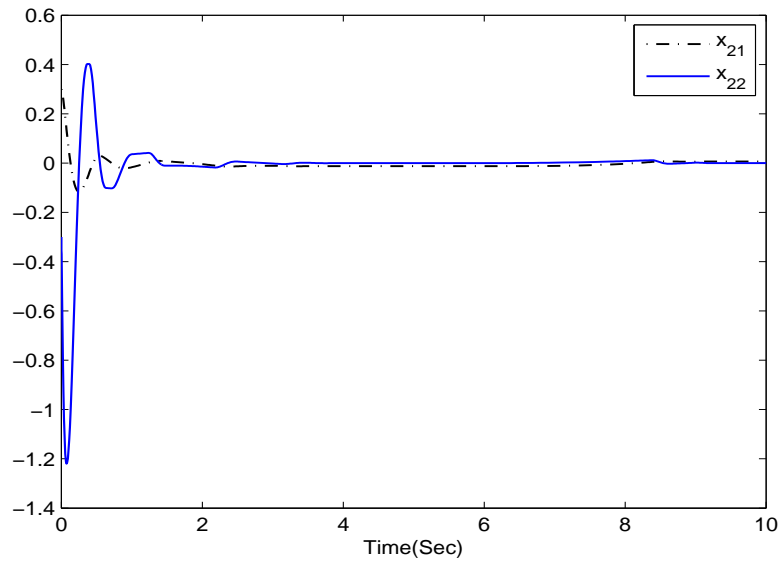


Fig. 2. State variables x_{21} and x_{22} .

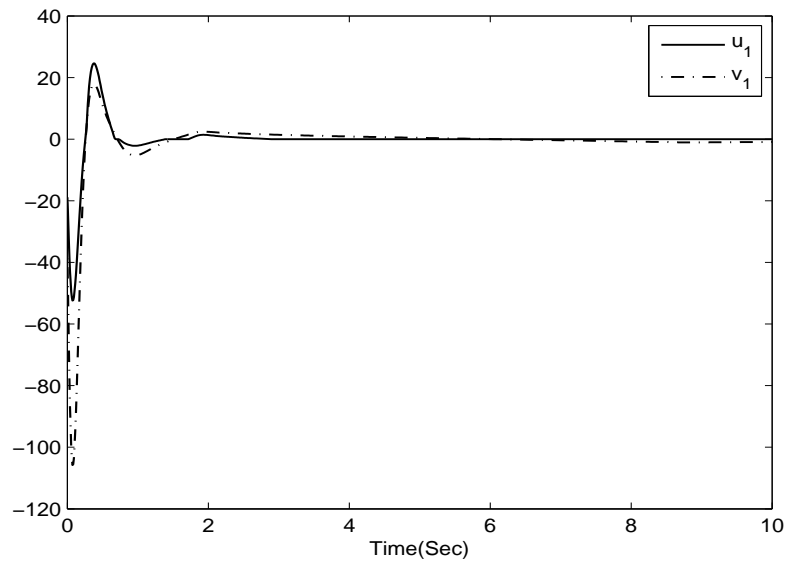


Fig. 3. The control input signals u_1 and v_1 .

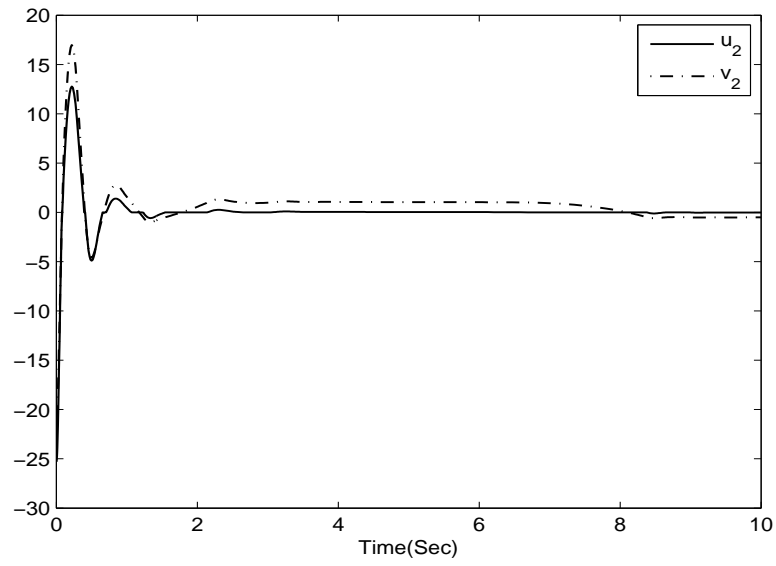


Fig. 4. The control input signals u_2 and v_2 .

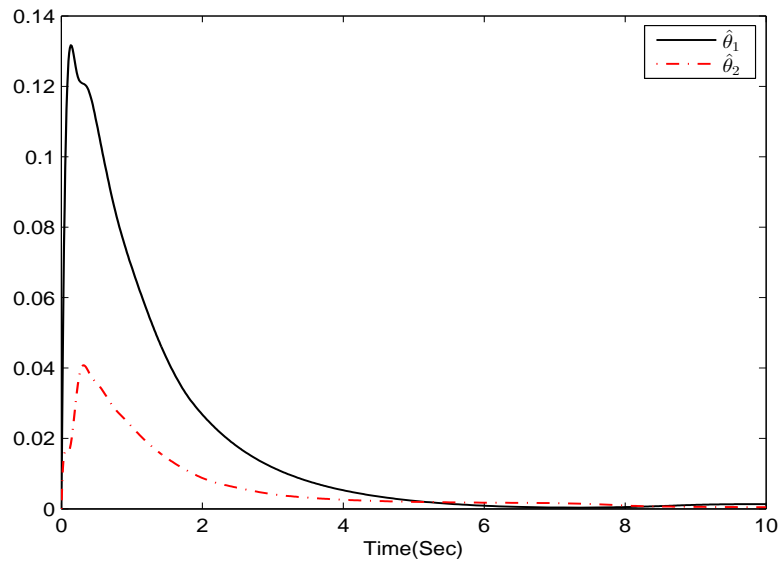


Fig. 5. The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.